Constraining a spatially dependent rotation of the Cosmic Microwave Background Polarization

Amit P.S. Yadav[1], Rahul Biswas[2], Meng Su[1], and Matias Zaldarriaga[1,3]

[1] *Center for Astrophysics, Harvard University, Cambridge, MA 02138, USA*
[2] *Department of Physics, University of Illinois at Urbana-Champaign, Urbana, IL 61801, USA and*
[3] *Department of Physics, Harvard University, Cambridge, MA 02138, USA*

Following Kamionkowski (2008), a quadratic estimator of the rotation of the plane of polarization of the CMB is constructed. This statistic can estimate a spatially varying rotation angle $\alpha(n)$. We use this estimator to quantify the prospects of detecting such a rotation field with forthcoming experiments. For PLANCK and CMBPol we find that the estimator containing the product of the E and B components of the polarization field is the most sensitive. The variance of this EB estimator, $N(L)$ is roughly independent of the multipole L, and is only weakly dependent on the instrumental beam. For FWHM of the beam size $\Theta_{fwhm} \sim 5' - 50'$, and instrument noise $\Delta_p \sim 5 - 50 \mu K$-arcmin, the scaling of variance $N(L)$ can be fitted by a power law $N(L) = 3.3 \times 10^{-7} \Delta_p^2 \Theta_{fwhm}^{1.3}$ deg^2. For small instrumental noise $\Delta_p \leq 5 \mu K$-arcmin, the lensing B-modes become important, saturating the variance to $\sim 10^{-6}$deg^2 even for an ideal experiment. Upcoming experiments like PLANCK will be able to detect a power spectrum of the rotation angle, $C^{\alpha\alpha}(L)$, as small as 0.01 deg^2, while futuristic experiment like CMBPol will be able to detect rotation angle power spectrum as small as 2.5×10^{-5} deg^2. We discuss the implications of such constraints, both for the various physical effects that can rotate the polarization as photons travel from the last scattering surface as well as for constraints on instrumental systematics that can also lead to a spurious rotation signal. Rotation of the CMB polarization generates B-modes which will act as contamination for the primordial B-modes detection. We discuss an application of our estimator to de-rotate the CMB to increase the sensitivity for the primordial B-modes.

PACS numbers:

I. INTRODUCTION

The polarization of the Cosmic Microwave Background (CMB) field can be studied in terms of the parity even E and parity odd B-modes [1, 2, 3, 4]. In standard cosmology, the physics governing the radiating field is parity invariant. Hence, the parity odd correlations $\langle TB \rangle$, $\langle EB \rangle$ vanish identically irrespective of the exact values of the cosmological parameters. However, the plane of linear polarization of CMB fields can be rotated due to interactions which introduce a different dispersion relation for the left and right circularly polarized modes, during propagation from the surface of last scattering surface to the earth. Such rotations generate non-zero cross-correlations $\langle TB \rangle$, $\langle EB \rangle$ in the CMB field. Thus, measurement of these correlations allows us to estimate the rotation of the plane of the CMB polarization [5]. Such interactions can come from three main sources: (a) interaction with dust foregrounds, (b) Faraday rotation due to interaction with background magnetic fields, and (c) interactions with pseudoscalar fields [6]. The interaction with foregrounds leads to a a frequency dependent effect. The same is true of Faraday rotation, where the frequency dependence is $(\sim \nu^{-2})$ [7, 8, 9, 10], while the interaction with pseudo-scalar fields is frequency independent. The distinct frequency dependencies allow one to separate these effects.

We know that parity is violated by weak interactions, and is possibly violated in the early universe, to give rise to baryon asymmetry. Hence, investigating the existence of parity violating interactions involving cosmologically evolving scalar fields is well motivated. As an example we consider an interaction of the form $\frac{\phi}{2M} F_{\mu\nu} \tilde{F}^{\mu\nu}$ [6, 11]. It has been shown that such a term can rotate polarization vector of linearly polarized light by an angle of rotation $\alpha = \frac{1}{M} \int d\tau \dot{\phi}$ during propagation for a conformal time τ. The fluctuations in the scalar field ϕ then will be imprinted in the rotation angle α of the polarization. It is interesting to ask what is the level of these fluctuations that can be detected with the upcoming CMB polarization experiments.

However, the observational situation is somewhat complicated by the fact that the measured CMB fields could be rotated with respect to the signal due to instrumental systematics: a mis-calibration of the orientation of the instrument which results in a constant rotation and differential offsets of the orientation of the individual detectors of the instrument resulting in rotation dependent on angular position $\hat{\mathbf{n}}$. It should be noted that this systematic effect is also a concern for the detection of polarization B modes, a major goal for forthcoming polarization experiments, since it can result in a spurious B-mode detection. Rotation of the CMB $\alpha(\mathbf{n})$, either due to interactions with a pseudoscalar field, or due to the instrumental rotation miscalibration can be a function of angular position $(\hat{\mathbf{n}})$.

An estimator for the spatially varying ration angle $\alpha(\hat{\mathbf{n}})$ has been reported in Kamionkowski [12]. Here, we point out that the widely used formalism for gravitational lensing [13] may be suitably modified to describe the rotation of CMB polarization. We use this to con-

struct approximate, but simpler form of the quadratic estimator of $\alpha(\hat{\mathbf{n}})$ using the flat-sky limit, and study its variance.

These estimators may be used to study the physics behind the rotation of the polarization of light. As we shall discuss, we can also put an upper-bound on the frequency independent rotation from non-standard interactions. We also discuss the use our the estimator $\hat{\alpha}(\hat{\mathbf{n}})$ to control instrumental rotation systematics for the detection of primordial B-modes. The presence of rotation systematics in the instrument generates B-modes. Hence, the control of rotation systematics of instruments is important for the measurement of B-modes of the CMB polarization; a major goal of subsequent polarization experiments. In this paper, we will discuss the prospects of detecting non-standard physics through measuring the angle of rotation, and the level to which we can control rotation systematics by this estimator.

A search for a constant rotation of the polarized light by an angle α from radio galaxies and the CMB is already underway [14, 15, 16, 17, 18, 19, 20, 21]. So far, there is no evidence of non-zero angle of rotation, and the angle α is constrained to be less than a few degrees [14, 15]. At present, there are no studies of constraints on spatially varying rotation angle $\alpha(\hat{n})$.

II. FORMALISM

In this section, we will construct an estimator for the spatially varying rotation field. We shall also describe a physical scenario which give rise to a frequency independent but spatially varying rotation.

A. Estimator for Spatially Dependent Rotation Field

We will describe the observable effect of rotation on the CMB polarization fields. Let the un-rotated (usual) CMB temperature field and the Stokes parameters at angular position $\hat{\mathbf{n}}$ be $\tilde{T}(\hat{\mathbf{n}})$, and $\tilde{Q}(\hat{\mathbf{n}})$, $\tilde{U}(\hat{\mathbf{n}})$ respectively. The relevant ensemble averages of the un-rotated CMB field can be encapsulated in

$$\langle \tilde{x}(\mathbf{l}) \rangle = 0, \qquad \langle \tilde{x}^\star(\mathbf{l}) \tilde{x}'(\mathbf{l}') \rangle = (2\pi)^2 \delta(\mathbf{l} - \mathbf{l}') \tilde{C}_l^{xx'}, \quad (1)$$

where x, x' run over the T, E, or B fields, and $\tilde{C}_l^{xx'}$ is the un-rotated CMB power spectrum. The temperature fields are invariant under a rotation of the polarization by an angle $\alpha(\hat{\mathbf{n}})$ at the angular position $\hat{\mathbf{n}}$, while the Stokes parameters transform like a spin two field. Thus, due to rotation, the observed fields are

$$(Q(\hat{\mathbf{n}}) \pm iU(\hat{\mathbf{n}})) = (\tilde{Q}(\hat{\mathbf{n}}) \pm \tilde{U}(\hat{\mathbf{n}})) \exp(\pm 2i\alpha(\hat{\mathbf{n}})). \quad (2)$$

The E and B fields of the CMB can be constructed from observed Stokes parameters. In a Fourier basis, in the

TABLE I: Minimum variance filters

xx'	$f_{xx'}(\mathbf{l}_1, \mathbf{l}_2)$
TE	$2\tilde{C}_{l_1}^{TE} \sin 2\varphi_{\mathbf{l}_1 \mathbf{l}_2}$
TB	$2\tilde{C}_{l_1}^{TE} \cos 2\varphi_{\mathbf{l}_1 \mathbf{l}_2}$
EE	$2[\tilde{C}_{l_1}^{EE} - \tilde{C}_{l_2}^{EE}] \sin 2\varphi_{\mathbf{l}_1 \mathbf{l}_2}$
EB	$2[\tilde{C}_{l_1}^{EE} - \tilde{C}_{l_2}^{BB}] \cos 2\varphi_{\mathbf{l}_1 \mathbf{l}_2}$
BB	$[\tilde{C}_{l_1}^{BB} + \tilde{C}_{l_2}^{BB}] \sin 2\varphi_{\mathbf{l}_1 \mathbf{l}_2}$

flat sky approximation,

$$[E \pm iB](\mathbf{l}) = \int d\hat{\mathbf{n}} \, [Q(\hat{\mathbf{n}}) \pm iU(\hat{\mathbf{n}})] e^{\mp 2i\varphi_{\mathbf{l}}} e^{-i\hat{\mathbf{l}} \cdot \hat{\mathbf{n}}}, (3)$$

where $\varphi_{\mathbf{l}} = \cos^{-1}(\hat{\mathbf{n}} \cdot \hat{\mathbf{l}})$. Since, the angle of rotation is already constrained to be small, we will work out the effects to first order in $\alpha(\hat{\mathbf{n}})$.

Since, we can only compute correlations of the CMB polarization modes theoretically, we want to isolate the change in correlations due to this rotation. Even in the absence of the physics causing rotation, we expect the CMB fields to be gravitationally lensed by matter inhomogeneities. Hence, the change due to rotation is the difference between lensed rotated fields, and lensed un-rotated fields $\tilde{T}, \tilde{E}, \tilde{B}$. We make the similarity of our problem with gravitational lensing of the CMB [13] manifest by writing the change in the CMB field modes $\delta x(l) = x(l) - \tilde{x}(l)$ due to rotation

$$\delta T(\mathbf{l}) = 0, \qquad (4)$$

$$\delta B(\mathbf{l}) = \int \frac{d^2 l'}{(2\pi)^2} \Big[\tilde{E}(\mathbf{l}') \cos 2\varphi_{\mathbf{l}'\mathbf{l}} - \tilde{B}(\mathbf{l}') \sin 2\varphi_{\mathbf{l}'\mathbf{l}} \Big] W(\mathbf{L}),$$

$$\delta E(\mathbf{l}) = -\int \frac{d^2 l'}{(2\pi)^2} \Big[\tilde{B}(\mathbf{l}') \cos 2\varphi_{\mathbf{l}'\mathbf{l}} + \tilde{E}(\mathbf{l}') \sin 2\varphi_{\mathbf{l}'\mathbf{l}} \Big] W(\mathbf{L}),$$

where $\mathbf{L} = \mathbf{l} - \mathbf{l}'$, and $\varphi_{\mathbf{l}\mathbf{l}'} = \varphi_{\mathbf{l}} - \varphi_{\mathbf{l}'}$ and $W(\mathbf{L}) = 2\alpha(\mathbf{L})$. Thus, due to rotation, a mode of wavevector $\mathbf{L}$ mixes the polarization modes of wavevectors $\mathbf{l}$ and $\mathbf{l}' = \mathbf{l} - \mathbf{L}$. Taking the ensemble average of the CMB fields for the fixed α field, one gets

$$\langle x^\star(\mathbf{l}) x'(\mathbf{l}') \rangle_{\text{CMB}} = \langle \tilde{x}^\star(\mathbf{l}) \tilde{x}'(\mathbf{l}') \rangle + f_{xx'}(\mathbf{l}, \mathbf{l}') \alpha(\mathbf{L}). \quad (5)$$

The TE, and TB correlations are produced indirectly via non-zero primordial TE correlation.

Our goal is to use Eq. (5) to construct a suitable estimator of the Fourier components $\alpha(\mathbf{L})$ of the rotation field in terms of the observed fields $T(\mathbf{l}), E(\mathbf{l}), B(\mathbf{l})$ and a theoretical computation of the power spectra involving un-rotated fields. Following Hu and Okamoto [13], we can define an unbiased quadratic estimator $\hat{\alpha}_{xx'}(\mathbf{L})$ for $\alpha(\mathbf{L})$ for each combination of the CMB modes $x(\mathbf{l}_1), x'(\mathbf{l}_2)$ by weighting quadratic combinations of dif-

ferent polarization modes by $F_{xx'}(\mathbf{l}_1, \mathbf{l}_2)$ appropriately:

$$
\begin{aligned}
\hat{\alpha}_{xx'}(\mathbf{L}) &= A_{xx'}(L) \int \frac{d^2 l_1}{(2\pi)^2} \big[x(\mathbf{l}_1) x'(\mathbf{l}_2) \\
&\quad - \langle \tilde{x}(\mathbf{l}_1) \tilde{x}'(\mathbf{l}_2) \rangle \big] F_{xx'}(\mathbf{l}_1, \mathbf{l}_2) ,
\end{aligned} \tag{6}
$$

where $\mathbf{L} = \mathbf{l}_2 - \mathbf{l}_1$, and the normalization

$$
A_{xx'}(L) = \left[\int \frac{d^2 l_1}{(2\pi)^2} f_{xx'}(\mathbf{l}_1, \mathbf{l}_2) F_{xx'}(\mathbf{l}_1, \mathbf{l}_2) \right]^{-1} , \tag{7}
$$

is chosen to make the estimator unbiased, i.e. $\langle \hat{\alpha}(\mathbf{L}) \rangle = \alpha(\mathbf{L})$. The fields $x(l)$ can be obtained from the map of an experiment, while the CMB power spectrum of un-rotated but lensed fields can be computed from publicly available Boltzmann codes like CMBfast and CAMB.

The weights $F_{xx'}$ can be optimized by minimizing the variance $\langle \tilde{\alpha}_{xx'}(\mathbf{L}) \tilde{\alpha}^\star_{xx'}(\mathbf{L}') \rangle$. For $xx' = TB$ and EB

$$
F_{xx'}(\mathbf{l}_1, \mathbf{l}_2) = \frac{f_{xx'}(\mathbf{l}_1, \mathbf{l}_2)}{C^{xx}_{l_1} C^{x'x'}_{l_2}} , \tag{8}
$$

where $C^{xx}_{l_2}$ and $C^{x'x'}_{l_2}$ are the observed power spectra including the effects of both the signal and the noise,

$$
C^{xx'}_\ell = \tilde{C}^{xx'}_\ell + C^{xx',n}_\ell , \tag{9}
$$

where $C^{xx',n}_\ell$ is the noise power spectrum. We assume the detector noise to be known apriori, be isotropic and Gaussian distributed. We include effects of beam-smearing by a symmetric Gaussian beam. Then, the noise power spectrum is

$$
C^{xx,n}_l = \Delta_x^2 e^{l^2 \Theta^2_{fwhm}/8 \ln 2}, \tag{10}
$$

where Δ_x is the instrument noise for temperature (x=T) or polarization (x=P); and Θ_{fwhm} is the full-width half-maximum (FWHM) resolution of the Gaussian beam. We will assume a fully polarized detector, for which $\sqrt{2}\Delta_T = \Delta_P$.

The variance of the estimator is

$$
\langle \tilde{\alpha}_{xx'}(\mathbf{L}) \tilde{\alpha}^\star_{xx'}(\mathbf{L}') \rangle = (2\pi)^2 \delta(\mathbf{L} - \mathbf{L}') \{ C^{\alpha\alpha}_L + N_{xx'}(L) \}, \tag{11}
$$

where for the minimum variance estimator, the Gaussian noise $N_{xx'}(L) = A_{xx'}(L)$, and gives the dominant contribution to the variance.

The Gaussian noise $N(L)$ is dependent only on the instrumental noise power spectrum $C^{xx'n}_l$ and the power spectrum of the un-rotated polarization field $\tilde{C}^{xx'}_l$. Hence the estimator noise depends on the cosmological parameters through the power spectrum of the polarization fields. We choose a standard fiducial model with a flat ΛCDM cosmology, with no rotation (i.e. $\alpha = 0$), with parameters described by the best fit to WMAP5 [14], given by $\Omega_b = 0.045, \Omega_c = 0.23, H_0 = 70.5, n_s = 0.96, n_t = 0.0,$ and $\tau = 0.08$.

Since, in reality the polarization field will be gravitationally lensed by the inhomogeneities in the matter distribution in the fiducial model, it is appropriate to use the power spectrum of lensed anisotropies as the un-rotated field in calculating $N(L)$. An angular remapping of photon positions due to gravitational lensing may result in an apparent frequency independent rotation of the plane of polarization, potentially biasing the estimator. Here, we show that the bias is negligible. Taking lensing into account the average of the estimator is

$$
\langle \hat{\alpha}_{xx'}(\mathbf{L}) \rangle_{CMB} =
$$
$$
\alpha(\mathbf{L}) + A_{xx'}(L) \int \frac{d^2 l_1}{(2\pi)^2} f^{lens}_{xx'} F_{xx'}(\mathbf{l}_1, \mathbf{l}_2) \phi^{len}(\mathbf{L}) , \tag{12}
$$

where ϕ^{len} is the line of sight projection of the gravitational potential $\Psi(\mathbf{x})$. The first term on the right hand side is the desired rotation field, and the second term represents the bias from lensing. The form of lensing filters $f^{lens}_{xx'}$ can be found in Table I of [13]. The lensing filter $f^{lens}_{xx'}$ are nearly orthogonal to the rotation window $F_{xx'}$. Hence the integrand of the lensing bias oscillates around zero, and even for $\phi^{len} \sim 1$, the bias is negligibly small in comparison to the square root of Gaussian noise $\sqrt{N(L)}$. Since Gaussian noise sets the minimum detectable rotation $\alpha(L)$, we can neglect the bias for all practical purposes.

B. Rotation from non-standard interactions

As discussed in introduction, apart from instrumental systematics, which we will discuss in section III B, a way of generating frequency independent rotation is by pseudoscalar fields coupled to photons.

There are no pseudoscalars in the standard model of particle physics that couple to radiation. However, they are common in particle physics beyond the standard model [22, 23, 24, 25]. In cosmology they have been invoked for dark matter or dark energy models, and also as solution to the fine-tuning problem of dark energy [26, 27, 28, 29, 30, 31].

A pseudoscalar field ϕ can couple to the electromagnetic fields by a Chern-Simons interaction term [11, 32, 38]

$$
\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} \partial_\mu \phi \partial^\mu \phi + \frac{\phi}{2M} F_{\mu\nu} \tilde{F}^{\mu\nu} , \tag{13}
$$

where $F_{\mu\nu}$ is the electromagnetic field strength tensor, $\tilde{F}^{\mu\nu}$ is its dual; and the pseudoscalar coupling with electromagnetic field is supressed by mass scale M.

The interaction term $\frac{\phi}{2M} F_{\mu\nu} \tilde{F}^{\mu\nu}$ is invariant under $U(1)$ gauge transformations, Lorentz symmetry and parity, and is suppressed by a mass scale M. The fact that no such effect has been detected in the laboratories puts a lower bound on M. It has been shown that such a term can rotate polarization vector of linearly polarized light

by an angle of rotation $\alpha = \frac{1}{M} \int d\tau \dot{\phi}$ during propagation for a conformal time τ [6], which is largest when the source of polarization is farthest, which happens for the CMB [5].

The angle of rotation along a line of sight depends on the change of ϕ along that line of sight. We can write the field ϕ in terms of a homogeneous piece $\phi_0(\tau)$ and a position dependent perturbation $\delta\phi(\hat{\mathbf{n}}, \tau)$. Then, the rotation angle of the CMB polarization

$$\alpha(\hat{\mathbf{n}}) = \frac{1}{M}\Delta\phi(\hat{\mathbf{n}}) = \frac{1}{M}\{\phi_0(\tau_0) - \phi_0(\tau_{dec}) - \delta\phi(\hat{\mathbf{n}}, \tau_{dec})\},$$
(14)

where τ_0 and τ_{dec} are the conformal times today and at the surface of last scattering respectively, and the perturbation $\delta\phi$ today, at the detector can be taken to be zero without any loss of generality. The shift symmetry of the Lagrangian implies that the field ϕ is classically massless. In our toy example we will assume that this was the case during inflation, and the quantum fluctuations during inflation were frozen in the field and will result in spatial fluctuations today. In this case we can write the power spectrum of fluctuations as nearly nearly scale invariant,

$$\langle\phi(\mathbf{k})\phi^\star(\mathbf{k}')\rangle = (2\pi)^3 P(k)\delta(\mathbf{k} - \mathbf{k}')$$
$$= (2\pi)^3 c_\phi k^{n_s-4}\delta(\mathbf{k} - \mathbf{k}')$$
(15)

with $n_s = 0.96$ [14], and $c_\phi = H^2/2$, where H is the Hubble parameter during the inflation. Computing the transfer function we find the power spectrum of the rotation angles to be

$$C_\ell^{\alpha\alpha} = \frac{2}{M^2\pi} \int k^2 dk P_\phi(k) j_\ell^2(kr)\Delta_\phi^2(k, \tau_{dec}),$$
(16)

where $r = \tau_0 - \tau_{dec}$ is the distance to the surface of last scattering, and $\Delta_\phi(k, \tau_{dec}) = 3j_1(k\tau_{rec})/k\tau_{dec}$ is the transfer function which is unity for scales larger than horizon size and decay with an oscillating envelop for modes inside the horizon.

We have discussed the scenario of physical interactions leading to rotation of the polarization field. There are two important physical parameters which determine the magnitude of the effect in this toy model: (a) the energy scale (or equivalently the Hubble rate) during inflation, which sets the amplitude of the fluctuation power spectrum of mass-less fields, and (b) The mass scale M by which the Chern-Simons interaction term is suppressed. The mass scale M is certainly much larger than the current energy scales of particle physics experiments $\sim 10^2$ GeV, and could be around or higher than the GUT scale $\sim 10^{16}$ GeV. In fact, the current best constraints on mass scale M come from the upper limit on B-modes, $M > 2.4 \times 10^{14} H_{14}$ GeV with $H_{14} = H/10^{14}$ [11]. The exact value of the Hubble scale during inflation is also unknown, with current estimates suggesting it to be around $\sim 10^{14} - 10^{15}$ GeV. For reference the tensor to scalar ratio

of perturbations produced in standard models of inflation is given by $r = 0.14 H_{14}^2$. This for current WMAP5 constraints [14] of $r < 0.2$ translates to $H_{14} < 1.2$. Since the potential of detection is linked to an assumed energy scale of inflation, and M, it is useful to write rotation angle in terms of H/M. The *rms* value of rotation angle for $L \lesssim 100$ can written as $\sqrt{\frac{L^2 C^{\alpha\alpha}(L)}{2\pi}} \sim \frac{10H}{M}$ deg. Our approach is to find the magnitude of the rotation which would be detectable.

III. RESULTS

A. Estimator

We discuss the prospects of using our unbiased estimator $\hat{\alpha}(\hat{\mathbf{n}})$ for detecting Fourier modes of the spatially varying rotation $\alpha(\hat{\mathbf{n}})$. We study the estimator variance and its dependence on instrumental characteristics focusing on planned missions.

We want to find out the magnitude of rotation that would be detectable. For this, the rotation signal must be larger than the noise in the estimator $N_{xx'}(L)$. In Fig. 1, we show the variance for the EB (red curves) and TB (blue curves) estimators [1] as a function of multipole L for the three experimental setups, (1) PLANCK satellite with noise level $\Delta_P = 56\mu K$-arcmin and FWHM of 7', (2) experiment with noise level $\Delta_P = 9.6\mu K$-arcmin and FWHM of 8', typical of upcoming ground and balloon-based CMB experiments (hereafter called Exp1), and (3) a CMBPol-like instrument with noise level $\Delta_P = \sqrt{2}\mu K$-arcmin and FWHM of 4', typical of future space-based CMB experiments. The black solid line shows the power spectrum of the cosmological rotation signal $C_l^{\alpha\alpha}$ from the Chern-Simon coupling discussed in section II B. We show this signal for $c_\phi/M^2 = 10^{-4}$, which corresponds to $M = 1.5 \times 10^{15} H_{14}$ GeV.

For the three experiments in consideration, the EB estimator is found to be the most sensitive, and the variance is roughly constant up to $L \sim 1000$. Although, we do not show $L = 0$ in the plot, our estimator can also be used to estimate the detectability of uniform rotation. For uniform rotation, $\mathbf{L} = \mathbf{l}_1 - \mathbf{l}_2 = 0$ in Eq. (7), hence there is no mode mixing between different wavevectors. The variance $N(L)$ for monopole and dipole are comparable; so rotation $\alpha(L)$ can be constrained to similar levels for these modes.

Currently only the constant angle of rotation (i.e. $L = 0$ of our estimator) has been constrained, $\alpha < 2^o$ [14, 15]; and at present there are no constraints on the spatial variation of the rotation angle $\alpha(\mathbf{n})$. From our Fig. 1 upcoming experiment like PLANCK will be able to

---

[1] For the three experiments considered in Fig. 1, the other estimators have much smaller sensitivity to the rotation angle.
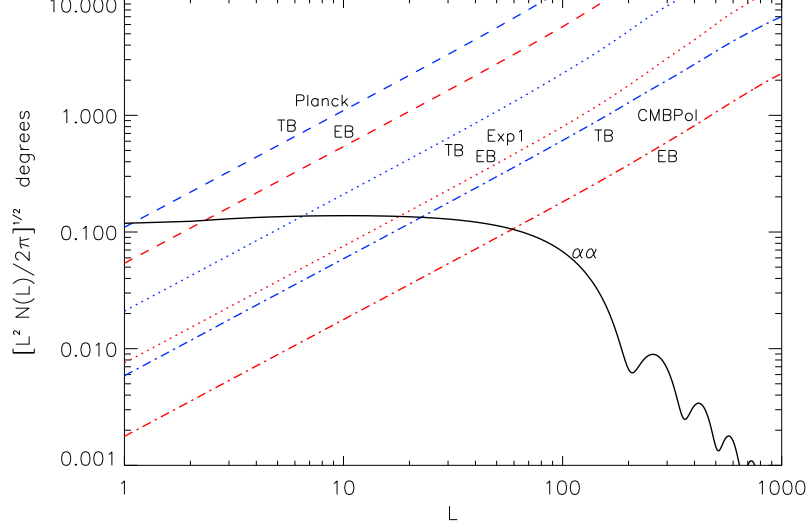
FIG. 1: Estimator variance $N(L)$ for the EB (red curves) and TB (blue curves) estimator as a function of multipole L. We show the noise for three experimental setups, CMBPol (dot-dashed), Exp1 (dot) and PLANCK (dashed). The solid black curve shows the rotation angle power spectrum for the model in which the pseudoscalar coupling to the electromagnetic field is suppressed by a mass scale M [11], and the perturbations in ϕ are seeded during the inflationary phase. We choose the fiducial value of the amplitude for power spectrum $c_\phi/M^2 = 10^{-4}$. This correspond to energy scale $M \sim 10^{15} H_{14}$ GeV, where H_{14} is the Hubble parameter during inflation in units of 10^{14} GeV. Note that with CMBPol like experiment, one is sensitive to energy scale as large as $M \sim 10^{17} H_{14}$ GeV, i.e. three orders of magnitude of improvement over the current best constraints.

detect the rotation angle power spectrum $C_\ell^{\alpha\alpha}$ as small as 0.01 deg^2, while futuristic experiment like CMBPol will be able to detect rotation angle power spectrum as small as 2.5×10^{-5} deg^2. These numbers translate to minimum detectible $H/M = 4 \times 10^{-3}$ for PLANCK, and $H/M = 2 \times 10^{-4}$ for CMBPol.

In Fig. 2, we study the dependence of the estimator variance with the detector noise Δ, and the beam size Θ. As in Fig. 1, here also we show the cosmological signal $C^{\alpha\alpha}(L)$ for reference, with the same fiducial parameter $c_\phi/M^2 = 10^{-4}$. The upper panel shows $N_{EB,EB}(L)$ as a function of L for various choices of instrumental noise Δ_p but for a fixed beam size of $4'$. The lower panel shows estimator noise $N_{EB,EB}(L)$ as a function of L for $\Delta_p = 2\mu K$-arcmin and for various choices of beam size, starting from $20'$ to $1'$. For FWHM of the beam size $\Theta_{fwhm} \sim 5'-50'$, and instrumental noise $\Delta_p \sim 5-50\mu K$-arcmin, the scaling of variance $N_{EB,EB}(L)$ can be fitted by a simple power law $N_{EB,EB}(L) = 3.3 \times 10^{-7} \Delta_p^2 \Theta_{fwhm}^{1.3}$ deg^2. However, we cannot minimize the estimator noise $N(L)$ to arbitrarily low levels by reducing the detector noise; the estimator variance plateaus out at a level of $\Delta_p < 5\mu K$-arcmin (larger than detector noise levels in CMBPol) to $\approx 10^{-6}$deg^2. Physically, this is due to lensing effects.

As discussed in Sec. II A, the estimator variance $N_{xx',xx'}(L)$ shown in Fig. 1 includes contribution due to lensing of E modes to B-modes. For experiments like Exp1 and PLANCK with $\Delta_p > 5\mu K$-arcmin, lens-

ing effects are negligible compared to the detector noise. Therefore, there is no difference in the estimator variance if the lensed polarization fields used as the un-rotated fields in Fig. 1, are replaced with un-lensed polarization fields. On the other hand, for experiments with $\Delta_p < 5\mu K$-arcmin (like CMBPol), lensing of the CMB power spectrum dominates the estimator variance, and eventually limits the sensitivity of the an idealized instrument to $\sim 10^{-6}$deg^2. Further, we calculated the leading order lensing contribution to noise, $N^{lens.}(L)$ which is proportional to lensing power spectrum $C^{len}(L)$, and is related to the connected part of the trispectrum. We find that this noise $N^{lens.}(L)$ is smaller than the estimator noise $N_{xx',xx'}(L)$ shown in Fig. 1 for all cases.

Can Estimator noise be further reduced? Lensing B-modes can be measured and hence, in principle, can be separated (de-lensing) from the pure rotation $\alpha(\hat{n})$ considered above. In the absence of lensing, the sensitivity of the idealized instrument would be limited by the cosmic variance of primordial B-modes. While de-lensing can improve the sensitivity of the idealized instrument, to the level of the noise inherent in the de-lensing process, it is likely to be challenging [33]. In Fig. 3 we show how much the Gaussian noise for the EB estimator $N(L)_{EB,EB}$ reduces as a function of amount of de-lensing. The de-lensed B-modes

$$B_\ell^{de-lens} = f B_\ell^{lens} \qquad (17)$$

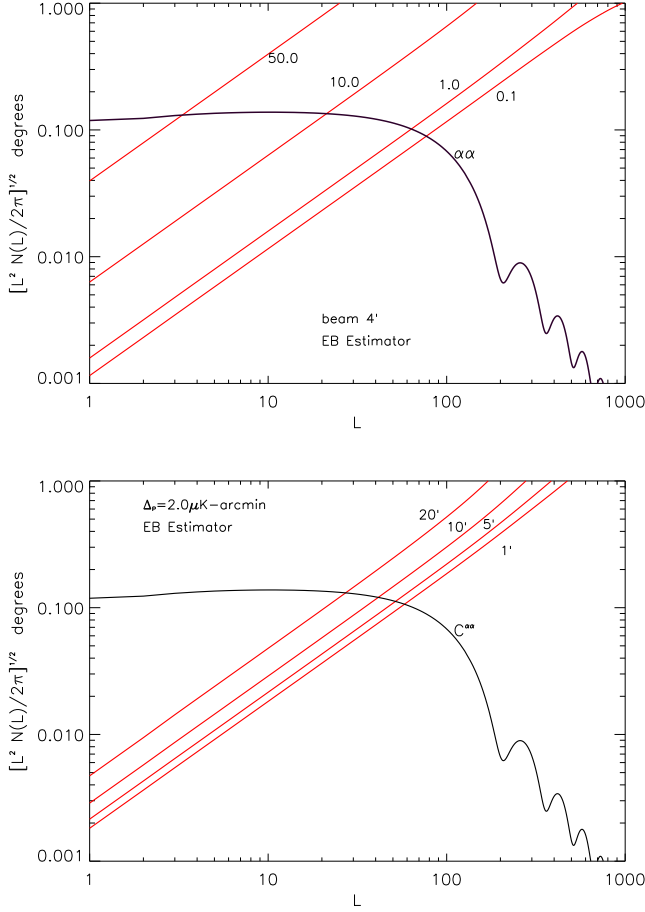are used in the estimator which reduces the variance, depending on the fraction of de-lensing f. The lower curve

FIG. 2: Dependence of the variance $N(L)$ of the EB estimator on instrumental characteristics as a function of multipole L: *Upper panel*: The diagonal lines represent the variance $N(L)$ for fixed FWHM of $4'$ but varying detector noise Δ_p. *Lower panel*: The diagonal lines represent the variance $N(L)$ for fixed detector noise of $\Delta_p = 2\mu K$-arcmin but varying the beam size. In both the panels the oscillatory curve represents the power spectrum of rotational field for the fiducial amplitude of $c_\phi/M^2 = 10^{-4}$.

B. Detectability of cosmological rotation and instrumental systematic effects

As indicated before, a cosmological rotation field $\alpha(\mathbf{n})$ can be confused with the instrumental rotation systematics. A calibration error in the angular position of the instrument is degenerate with a spatially constant angle of rotation (L=0 of $\alpha(\mathbf{L})$), while errors in the rota-
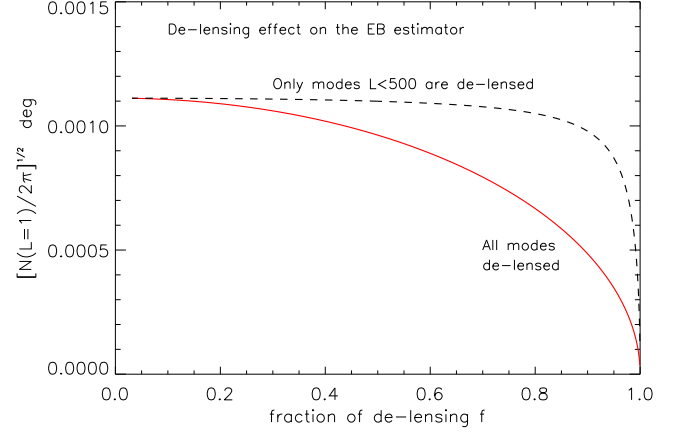

FIG. 3: Gaussian noise (at L=2) for the EB estimator as a function of fraction of De-lensing of the CMB. Since the lensing effects are only important for experiments with $\Delta_p < 5\mu K$-arcmin, we assume an idealized experiment (i.e. $\Delta = 0, \Theta_{fwhm} = 0$) to see the effect of de-lensing. Lower (solid red) curve assumes that all the observed B-modes ($l_{max} = 3000$) are being de-lensed by equal amount. Upper (dash black) curve shows the effect of de-lensing when only the modes with $\ell < 500$ are being de-lensed, while no de-lensing for $\ell > 500$.

tion calibration of individual detectors in the instrument, leading to a relative mis-alignment of axes of the individual detectors by angles ω_i are degenerate with spatially varying $\alpha(\hat{\mathbf{n}})$. For an instrument with a large number of detectors, we can treat the angles of rotation of the ith detector ω_i as a smooth field as a function of the detector position. The relative offsets in the polarimeters in the detector could result in a systematics signal $\omega(\mathbf{n})$ in the map if the weighting of each polarimeter changes from pixel to pixels in the map. This depends on the scan strategy. For illustration purposes we can model the statistical properties of systematics signal [34, 35] as a statistically isotropic Gaussian field with a power spectrum given by

$$C_l^{\omega\omega} = \frac{A_\omega^2 \exp(-l(l+1)\sigma_\omega^2)}{\int \frac{d^2l}{(2\pi)^2} \exp(-l(l+1)\sigma_\omega^2)}, \qquad (18)$$

where A_ω characterizes the *rms* value of this field ω, and σ_ω is a coherence length.

To see the effect of rotation systematic field, we can change $\alpha(\hat{\mathbf{n}})$ in Eq. (2) to $\alpha(\hat{\mathbf{n}}) + \omega(\hat{\mathbf{n}})$, and re-derive our estimator. The systematics field biases our estimator $\hat{\alpha}(\mathbf{L})$ by an amount $\omega(\mathbf{L})$ and increases the variance of the xx' estimator by an amount $C^{\omega\omega}(L) + N_{xx',xx'}^{\omega\omega}(L)$ (see appendix). For our model of systematics field and assuming that α and ω fields are uncorrelated, the power spectrum of bias is given by Eq. (18) i.e. $C^{\omega\omega}(L)$. For the expression for the systematic noise $N_{EB,EB}^{\omega\omega}(L)$ please refer to the appendix. In order to use the estimator for detection of a rotation field with power spectrum $C^{\alpha\alpha}(L)$,
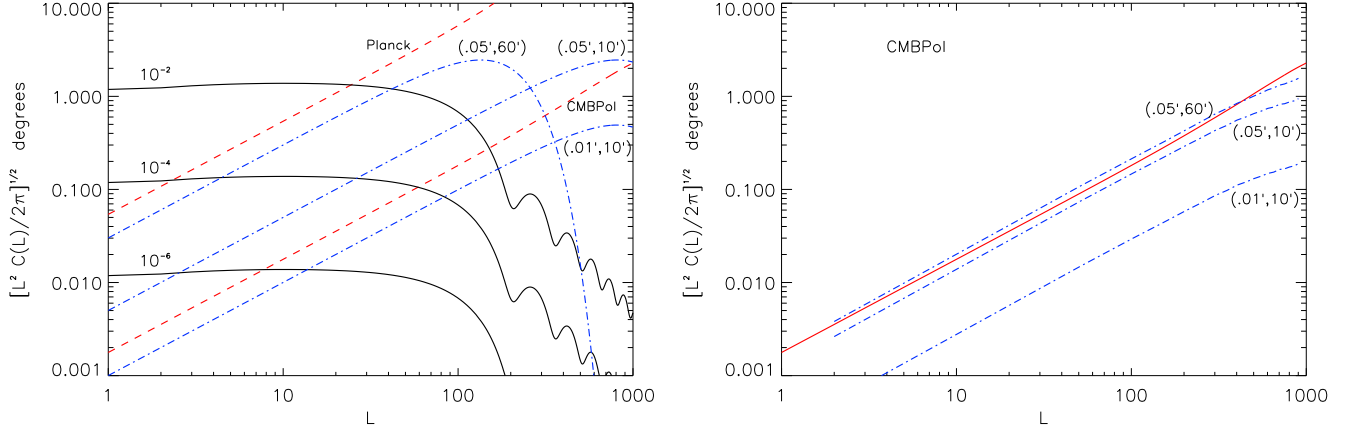
(solid red) shows the most optimistic scenario where all the CMB modes are being de-lensed (up to $L = 3000$). The upper curve shows, although still challenging, a more realistic case where only the modes $L < 500$ are being de-lensed.

FIG. 4: *Left Panel*: Dot dashed (blue) curves show the systematics rotation power spectrum $C^{\omega\omega}(L)$ for three combinations of *rms* amplitude A_ω (in arcmin) and FWHM of coherence length, $\sqrt{8\ln(2)}\sigma_\omega$ (in arcmin). The two dashed red curves show the Estimator variance $N_{EB,EB}(L)$ for the PLANCK (upper) and CMBPol (lower) experiment. The black oscillatory curve shows the power spectrum of rotational field for the fiducial amplitude of $c_\phi/M^2 = 10^{-2}, 10^{-4}$, and 10^{-6}. *Right Panel*: For CMBPol experiment, solid red curve shows the estimator variance $N_{EB,EB}(L)$ and the dot dashed (blue) curves show the systematics noise $N^{\omega\omega}_{EB,EB}(L)$ for three combinations of *rms* amplitude A_ω (in arcmin) and FWHM of coherence length, $\sqrt{8\ln(2)}\sigma_\omega$ (in arcmin) as the left panel.

$C^{\alpha\alpha}(L) >> C^{\omega\omega}(L)$.

In order for the cosmological rotation field $\alpha(\hat{n})$ to be determined to the noise levels in Fig. 1, both the systematic bias power spectrum $C^{\omega\omega}(L)$ and systematic noise $N^{\omega\omega}_{xx',xx'}(L)$ should be smaller than the estimator noise $N_{xx',xx'}(L)$. In Fig. 4, we study the dependence of the systematic field power spectrum $C^{\omega\omega}(L)$ and the systematics noise term $N^{\omega\omega}_{EB,EB}(L)$.

In the left panel of Fig. 4 the dot dashed (blue) curves show the systematics rotation signal for various choices of *rms* amplitude A_ω and FWHM of coherence length $\sqrt{8\ln(2)}\sigma$. The systematics signal does not depend on the instrumental noise Δ, and beam, Θ_{fwhm}. Solid (black) curves show the fiducial cosmological rotation power spectrum $C^{\alpha\alpha}(L)$ for $c_\phi/M^2 = 10^{-2}, 10^{-4}, \& 10^{-6}$, and the two dashed (red) curves show the Gaussian noise for the EB estimator $N_{EB,EB}(L)$ for PLANCK (upper) and CMBPol (lower) experiments.

In right panel of Fig. 4 the dot dashed (blue) curves show the systematics noise for EB estimator $N^{\omega\omega}_{EB,EB}(L)$ for the CMBPol like experiment and for various choices of *rms* amplitude A_ω and coherence length σ. The solid (red) line shows the Gaussian noise for the EB estimator $N_{EB,EB}(L)$ for CMBPol experiment.

Note that like the Gaussian noise $N_{EB,EB}(L)$, the systematic noise $N^{\omega\omega}_{EB,EB}(L)$ is also weakly dependent on L. However the systematic power spectrum (depending on the coherence length) may only be similarly flat only to about $L \sim 100$ (for coherence $60'$). Hence, we may characterize their values by their values at a particular value of $L < 100$. Together, the two panels show that at a coherence length of $10'$, an *rms* amplitude of $\approx 0.01'$ gives

a systematic variance which is about ten times smaller the estimator variance in the CMBPol experiment, while the bias power spectrum is only about half the estimator variance. Smaller coherence length σ and *rms* amplitude of the systematic fields result in smaller effect of the rotation systematics. This implies, that in order to detect rotation, using the CMBPol experiment, one would have to control the systematic field to much better than $\{\sigma_\omega, A_\omega\}$ values $\{10', 0.01'\}$.

C. De-rotating CMB to improve the sensitivity for the Primordial B-modes Detection

An important design goal of the futuristic CMB polarization experiments is the detection of primordial B-modes. Rotation generates B-modes (via Eq. (4)) which can be confused with the primordial B-modes. Both the instrumental rotation systematics and any cosmological rotation will generate B-modes. Hence, in order to study primordial B-modes, it is necessary to know the level of these spurious B-modes.

An important application of our estimator is to measure rotation and then in turn de-rotate the CMB polarization field to remove the spurious B-modes and hence increase sensitivity to the primordial B-modes detection. For this application, it is not important to know what the source of this rotation is.

A specific example is the case when cosmological rotation is known (or assumed) to be small and we are interested in controlling instrumental systematics to detect primordial B-modes. One can in this case layout specifics
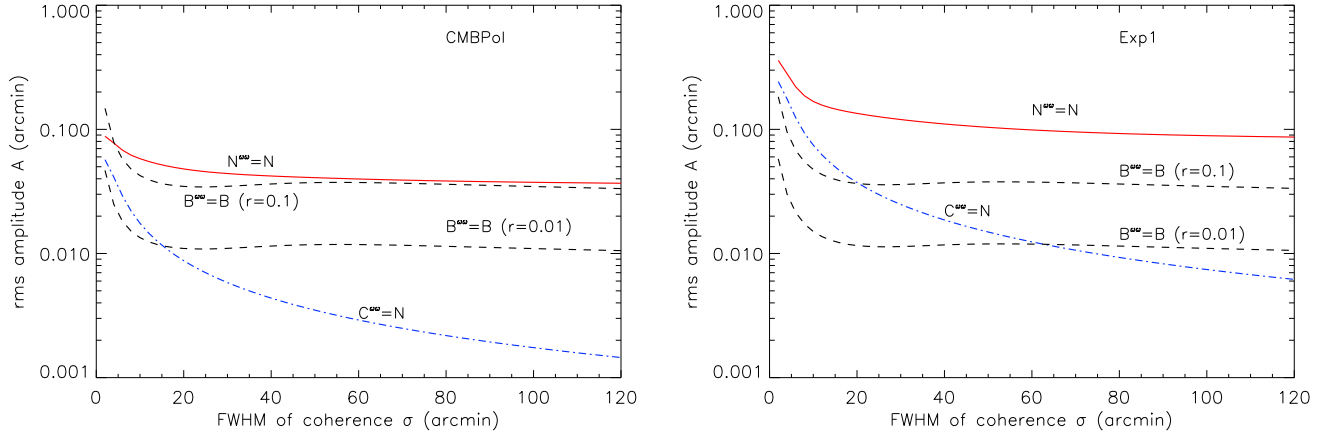
FIG. 5: The requirement on control of the systematic fields ω for CMBPol (*Left Panel*), and Exp1 (*Right Panel*) for the detection of primordial B-modes, and the level to which the CMB can be de-rotated.. The dashed black line shows the systematic parameters $\{A, \sigma\}$ which generate spurious B-modes of the same magnitude at ($L = 90$) as the primordial B-modes (at L=90, where the primordial B-modes peak); for values of $r = 0.1$ (upper back) and $r = 0.01$ (lower black). The solid red curve represents the systematic fields for which the variance contribution due to the systematic field $N^{\omega\omega}_{EB,EB}(L = 2)$ becomes equal to the estimator variance $N_{EB,EB}(L = 2)$. The blue line represents the systematic fields for which the power spectrum of the ω field at $L = 2$ is equal to the estimator variance $N_{EB,EB}(L = 2)$.

on what is the minimum B-modes amplitude that will be detectable without worrying about rotation systematics. The amplitude of the primordial B-modes is fixed by amplitude of tensor perturbations which depends on the energy scale of inflation. Equivalently one can use the ratio of amplitude of tensor and scalar perturbations r to characterize the B-modes.

In Fig. 5 we show the comparison of required control of the systematics ω for CMBPol (left panel), and Exp1 (right panel) for the detection of primordial B-modes, and the level to which the CMB can be de-rotated. The dashed black lines show the A_ω, and σ_ω of the systematic fields (see Eq. (18)) for which spurious B-mode power spectrum $C^{\omega\omega}(L)$ at $L = 90$ is equal the primordial B-modes power spectrum (at L=90) for values of $r = 0.1$ (upper curve), and $r = 0.01$ (lover curve). The solid red curve shows the A_ω, and σ_ω of the systematic field for which the variance contribution due to the systematic field $N^{\omega\omega}_{EB,EB}(L)$ at $L = 2$ becomes equal to the Gaussian noise $N_{EB,EB}(L)$ at $L = 2$. The dot-dashed blue curve shows the A_ω, and σ_ω of the systematic field for which the power spectrum of the ω field $C^{\omega\omega}(L)$ at $L = 2$ is equal to the estimator variance $N^{\omega\omega}_{EB,EB}(L)$ at L=2.

In order to be able to detect B-modes of a particular r value, the parameters $\{A, \sigma\}$ must be in the region below the corresponding black lines. Dot-dashed blue curves represent the level of rotation systematics to which our estimator can be used to correct for; i.e. if the systematics, $\{A, \sigma\}$ are above the blue curve then our estimator can reduce it to the level given by blue curve. If the instrumental systematics parameter $\{A, \sigma\}$ are above the red curve, our estimator is no longer a minimum variance

estimator, and one should include the systematics noise effect $N^{\omega\omega}$ in the variance of our estimator. However if instrumental systematics parameter $\{A, \sigma\}$ are below the red curve, then for a given r if the systematics requirement for B-mode is less stringent that the requirement from rotation, one can use our estimator to control the rotation systematics for B-modes detection.

If the systematics parameters $\{A, \sigma\}$ are below the dot-dashed blue curve, then the rotation from the systematics is smaller than the sensitivity of the estimator. If our estimator detects rotation signal in this case then it can be attributed to cosmological signal or to incorrectness of the systematics model.

IV. SUMMARY AND DISCUSSION

There are interesting physical mechanisms that can rotate the plane of CMB polarization. We presented explicit formulae for estimators of the spatially varying rotation angle $\alpha(\hat{\mathbf{n}})$ that can be constructed from future datasets in the flat sky limit. By computing the variance of these estimators, we estimate how large a variation in the angle α must be to be detected by a particular experiment. Currently only the constant angle of rotation (i.e. $L = 0$ of our estimator) has been constrained, $\alpha < 2^o$ [14, 15]. At present there are no constraints on the spatial variation of the rotation angle $\alpha(\mathbf{n})$. From our Fig. 1 upcoming experiment like PLANCK will be able to detect the rotation angle power spectrum $C^{\alpha\alpha}_\ell$ as small as 0.01 deg^2, while futuristic experiment like CMBPol will be able to detect rotation angle power spectrum as

small as 2.5×10^{-5} deg^2. These numbers translate to minimum detectible $H/M = 4 \times 10^{-3}$ for PLANCK, and $H/M = 2 \times 10^{-4}$ for CMBPol.

Gravitational lensing does not bias the estimator, however it increase the variance of the estimator. The increase in the variance is sub-dominant for experiments with $\Delta_p > 5\mu K$-arcmin. For small instrumental noise $\Delta_p \leq 5\mu K$-arcmin, the lensing B-modes become important, saturating the variance to $\sim 10^{-6}$deg^2 even for an ideal experiment.

The physical mechanisms that give rise to the rotation field all probe interesting cosmological physics; in principle, they can be separated using their frequency dependence and used to study the magnetic field or polarization dust maps. A cosmological source for a frequency independent pure rotation field can be interpreted as exotic feature signifying a clear departure from standard model physics. Such a departure could be a violation of the equivalence principle, or violation of Lorentz invariance [36, 37]. In the context of cosmology, an important example is pseudoscalar fields which have been proposed as dynamical models of dark energy to solve the cosmological constant problem, dark matter, and also as a solution to the fine tuning problem of dark energy.

We have considered a model in which the perturbations in the scalar field were imprinted during inflation and the scalar field couples to photon via Chern-Simons coupling discussed in Sec. II B, which are suppressed by mass scale M. With CMBPol like experiment one can constrain the mass scale $M > 10^{17}H_{14}$ GeV, where H_{14} is the Hubble parameter during inflation in the units of 10^{14} GeV.

If the scalar field is assumed to be responsible for the dark energy, the constant rotation would probe the coupling scales of such a dark energy field, and establish its dynamic nature. The spatially varying part of this rotation field would also probe the clustering of such a field. Typically, such fields would have large sound speeds, so that clustering is only possible at large scales. Hence, the possibility of detection of the spatially varying field is best at low multipoles. If one assumes that the field

has the kind of clustering discussed in [38], then from an experiment like CMBPol, one can constrain the mass scale of suppression of the Chern-Simons coupling term, to $M \gtrsim 10^{10}$ GeV.

However, the rotation field induced by a cosmological source can be degenerate with the rotation systematics of the instrument, which are limited by the rotation calibration of the polarimeters. Thus, the detection of such cosmological signals is only possible if the cosmological signal is larger than the level of rotation systematics signal that can be controlled. Rotation systematics also effect the variance of our estimator. We quantify the level of systematics control required for detection of the cosmological signal to be only limited by the estimator noise $N(L)$. If one can show from other experiments that the sources of such cosmological signals can be limited to magnitudes $|\delta\alpha(\mathbf{L})|$ smaller than these systematic levels, then we cannot detect the cosmological signals. However, then we can use this fact that the observed rotation field should be less than this magnitude to calibrate the instrument to control the level of rotation systematics to precision levels of $\sim |\delta\alpha(\mathbf{L})|$. This could enable a better study of effects like primordial B-modes, lensing, or the frequency dependent signals from sources like magnetic fields or foreground dust. Thus precise studies of this rotation field could either probe exciting physical effects, and/or enable better control of calibration and instrumental statistics.

[1] U. Seljak and M. Zaldarriaga, Phys. Rev. Lett. **78**, 2054 (1997).
[2] U. Seljak and M. Zaldarriaga (1998), astro-ph/9805010.
[3] M. Kamionkowski, A. Kosowsky, and A. Stebbins, Phys. Rev. D **55**, 7368 (1997).
[4] M. Kamionkowski, A. Kosowsky, and A. Stebbins, Phys. Rev. Lett. **78**, 2058 (1997).
[5] A. Lue, L. Wang, and M. Kamionkowski, Phys. Rev. Lett. **83**, 1506 (1999), arXiv:astro-ph/9812088.
[6] S. M. Carroll, Phys. Rev. Lett. **81**, 3067 (1998).
[7] A. Kosowsky and A. Loeb, Astrophys. J. **469**, 1 (1996).
[8] A. Kosowsky, T. Kahniashvili, G. Lavrelashvili, and B. Ratra, Phys. Rev. D **71**, 043006 (2005).
[9] L. Campanelli, A. D. Dolgov, M. Giannotti, and F. L. Villante, Astrophys. J. **616**, 1 (2004).
[10] C. Scóccola, D. Harari, and S. Mollerach, Phys. Rev. D **70**, 063003 (2004), arXiv:astro-ph/0405396.
[11] M. Pospelov, A. Ritz, and C. Skordis (2008), 0808.0673.
[12] M. Kamionkowski, ArXiv e-prints (2008), 0810.1286.
[13] W. Hu and T. Okamoto, Astrophys. J. **574**, 566 (2002).
[14] E. Komatsu et al. (WMAP) (2008), 0803.0547.
[15] . E. Y. S. Wu et al. (QUaD) (2008), astro-ph/0811.0618.
[16] J.-Q. Xia, H. Li, G.-B. Zhao, and X. Zhang, Astrophys. J. Lett. **679**, L61 (2008), 0803.2350.
[17] J.-Q. Xia, H. Li, X. Wang, and X. Zhang, Astronomy and Astrophysics **483**, 715 (2008), 0710.3325.
[18] B. Feng, M. Li, J.-Q. Xia, X. Chen, and X. Zhang, Phys. Rev. Lett. **96**, 221302 (2006), arXiv:astro-ph/0601095.
[19] T. Kahniashvili, R. Durrer, and Y. Maravin, Phys. Rev. D **78**, 123009 (2008).
[20] S. M. Carroll, G. B. Field, and R. Jackiw, Phys. Rev. **D41**, 1231 (1990).

[21] P. Cabella, P. Natoli, and J. Silk, Phys. Rev. D **76**, 123014 (2007), 0705.0810.
[22] R. D. Peccei and H. R. Quinn, Phys. Rev. Lett. **38**, 1440 (1977).
[23] F. Wilczek, Phys. Rev. Lett. **40**, 279 (1978).
[24] S. Weinberg, Phys. Rev. Lett. **40**, 223 (1978).
[25] R. Battesti, B. Beltrán, H. Davoudiasl, M. Kuster, P. Pugnat, R. Rabadán, A. Ringwald, N. Spooner, and K. Zioutas, in *Lecture Notes in Physics, Berlin Springer Verlag*, edited by M. Kuster, G. Raffelt, and B. Beltrán (2008), vol. 741 of *Lecture Notes in Physics, Berlin Springer Verlag*, pp. 199–+.
[26] J. A. Frieman, C. T. Hill, A. Stebbins, and I. Waga, Phys. Rev. Lett. **75**, 2077 (1995), astro-ph/9505060.
[27] K. Freese, J. A. Frieman, and A. V. Olinto, Phys. Rev. Lett. **65**, 3233 (1990).
[28] C. Wetterich, Nucl. Phys. **B302**, 668 (1988).
[29] N. Kaloper and L. Sorbo, JCAP **0604**, 007 (2006).
[30] K. Dutta and L. Sorbo, Phys. Rev. **D75**, 063514 (2007).
[31] A. Abrahamse, A. Albrecht, M. Barnard, and B. Bozek, Phys. Rev. **D77**, 103503 (2008), 0712.2879.
[32] S. M. Carroll, Phys. Rev. Lett. **81**, 3067 (1998).
[33] K. M. Smith et al. (2008), 0811.3916.
[34] W. Hu, M. M. Hedman, and M. Zaldarriaga, Phys. Rev. **D67**, 043004 (2003).
[35] M. Su, A. P. S. Yadav, and M. Zaldarriaga (2009), astro-ph/0901.0285.
[36] D. Colladay and V. A. Kostelecky, Phys. Rev. **D58**, 116002 (1998).
[37] A. Kostelecky and M. Mewes (2007), astro-ph/0702379.
[38] M. Li and X. Zhang, ArXiv e-prints (2008), 0810.0403.

APPENDIX A: CONTAMINATION

Instrumental rotation systematics and lensing of the CMB can effect our estimator. In this appendix we will show how the rotation systematics and lensing of CMB appear in our estimator. Let us denote systematics field by $\omega(L)$ and lensing filed by $\phi^{len}(L)$. We can incorporate the effect of rotation systematics by changing $\alpha(\hat{\mathbf{n}})$ to $\alpha(\hat{\mathbf{n}}) + \omega(\hat{\mathbf{n}})$, and effect of lensing by changing $\hat{\mathbf{n}}$ to $\hat{\mathbf{n}} + d(\hat{\mathbf{n}})$ in Eq. (1), where $\mathbf{d} = \nabla\phi^{len}$. In the presence of rotation systematics and lensing, the average of the estimator is given as

$$\langle \hat{\alpha}_{EB}(\mathbf{L}) \rangle = \alpha(\mathbf{L}) + \omega(\mathbf{L}) + A_{EB}(L) \int \frac{d^2 l_1}{(2\pi)^2} f_{EB}^{len} F_{EB} \phi(L), \tag{A1}$$

where the first term on the right hand side is the desired rotation field, the second term is the estimator bias from instrumental systematics, and the third term represents the bias from lensing. The variance of the estimator can

be written as

$$\left\langle \left\langle \langle \hat{\alpha}_{EB}(\mathbf{L}) \cdot \hat{\alpha}_{EB}(\mathbf{L}') \rangle_{\text{CMB}} \right\rangle_{\text{LSS}} \right\rangle_{\text{SYS}}$$
$$= A_{EB}(L) A_{EB}(L') \times$$
$$\int \frac{d^2 l_1}{(2\pi)^2} \int \frac{d^2 l_{1'}}{(2\pi)^2} F_{EB}(\mathbf{l}_1, \mathbf{l}_2) F_{EB}(\mathbf{l}_1{}', \mathbf{l}_2{}')$$
$$\times \left\{ \langle E(\mathbf{l}_1)^{\text{obs}} B(\mathbf{l}_2)^{\text{obs}} E(\mathbf{l}_1')^{\text{obs}} B(\mathbf{l}_2')^{\text{obs}} \rangle \right\}$$
$$= (2\pi)^2 \delta_D(\mathbf{L} + \mathbf{L}') \times$$
$$\left[C^{\alpha\alpha}(L) + C^{\omega\omega}(L) + N_{EB,EB}(L) + N_{EB,EB}^{\alpha\alpha}(L) \right.$$
$$\left. + N_{EB,EB}^{\omega\omega}(L) + N_{EB,EB}^{(len)}(L) + ... \right], \tag{A2}$$

where $\mathbf{L} = \mathbf{l}_1 + \mathbf{l}_2$, $C^{\alpha\alpha}(L)$ is the cosmological rotation power spectrum, and $C^{\omega\omega}(L)$ is the rotation systematics power spectrum. The terms $N_{EB,EB}(L)$, $N_{EB,EB}^{\alpha\alpha}(L)$, $N_{EB,EB}^{\omega\omega}(L)$, and $N_{EB,EB}^{(len)}(L)$ are the estimator Gaussian noise, the first order non-Gaussian estimator noise, the first order systematics noise of instrumental rotation, and the first order lensing induced non-Gaussian noise, respectively. Like the lensing quadratic estimator, the Gaussian noise comes from the disconnected part of the four-point function, while non-Gaussian noise $N_{EB,EB}^{(\alpha)}$, $N_{EB,EB}^{(\omega)}$, and $N_{EB,EB}^{(len)}$ comes from the connected part. We note that the Gaussian noise term also includes rotation systematic effects implicitly since instrumental systematics bias the measured rotation power spectrum with $C^{\omega\omega}(L)$ with as we have shown, note that we assume no cross correlation term $C^{\alpha\omega}(L)$. The ellipses in Eq. (A2) stands for higher order terms. The first order non-Gaussian noise can be written as

$$N_{EB,EB}^{(X)}(L) =$$
$$A_{EB}^2(L) \int \frac{d^2 l_1}{(2\pi)^2} \int \frac{d^2 l_2}{(2\pi)^2} F_{EB}(\mathbf{l}_1, \mathbf{l}_2) F_{EB}(\mathbf{l}_1{}', \mathbf{l}_2{}') \times$$
$$\left[C_{l_1}^{EE} C_{l_1'}^{EE} \left\{ C_{|\mathbf{l}_1+\mathbf{l}_2|}^{XX} W_B^X(\mathbf{l}_2, -\mathbf{l}_1) W_B^X(\mathbf{l}_2', -\mathbf{l}_1') \right. \right.$$
$$\left. \left. + C_{|\mathbf{l}_1+\mathbf{l}_2'|}^{XX} W_B^X(\mathbf{l}_2, -\mathbf{l}_1') W_B^X(\mathbf{l}_2', -\mathbf{l}_1) \right\} \right], \tag{A3}$$

where $X = \{\alpha, \omega, \phi^{len}\}$, the window functions $W_B^\omega(\mathbf{l}_1, \mathbf{l}_2) = W_B^\alpha(\mathbf{l}_1, \mathbf{l}_2) = 2\cos[2(\varphi_{l_2} - \varphi_{l_1})]$, and $W_B^{\phi^{len}}(\mathbf{l}_1, \mathbf{l}_2) = \sin 2(\varphi_{\mathbf{l}_1} - \varphi_{\mathbf{l}_2})(\mathbf{L} \cdot \mathbf{l}_1)$. We use this equation to numerically compute the systematic-induced estimator noise for the rotation systematics. Among these extra covariance noise, $N_{EB,EB}^{(\alpha)}$, and $N_{EB,EB}^{(L)}$ are always smaller than the estimator Gaussian noise $N_{EB,EB}(L)$, and $N_{EB,EB}^{(\omega)}$ can be in some cases comparable to $N_{EB,EB}(L)$. We use Fig. 5 to illustrate when

$N_{EB,EB}^{(\omega)}$ goes to the same level as $N_{EB,EB}(L)$ under certain experiment configuration.